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HYDRODYNAMIC INSTABILITY OF THE ABLATION FRONT IN THE PRESENCE OF ABLATION ACCELERATION OF A LAYER

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UDC 532.5+533.95

1. A large number of papers on the instability of an ablation front (AF) accompanying the acceleration of a layer by the ablation pressure has now been published [1-13]. The Cauchy problem, linearized around the stationary flow, which is found by numerical calculation, is studied numerically in [4, 5]; the numerical calculation of the Cauchy problem, linearized around the stationary flow, is carried out in [6, 7]. The stationary solution is found by numerical integration of a system of ordinary differential equations. We must make an important remark regarding [6-7]. We shall show that the stationary flow in a gravitational field has a peculiarity which invalidates the results of [6-7], regarding taking into account of the compressibility of the cold material and of the long-wavelength perturbations. We shall study the stationary solution in the region filled with cold matter. In this region, in the vicinity of the AF the flow is subsonic ($M \ll 1$). In the presence of gravity, the Mach number $M = v/c$ in the subsonic flow increases monotonically away from the AF in the cold matter and at some distance L_1 from the AF $M = 1$. The point is that in the cold matter the electronic thermal conductivity is small and the heat fluxes correspond as negligibly small. Therefore the stationary flow of cold matter is isentropic. For subsonic flow with $M \ll 1$ in the vicinity of the AF, because of the effect of the gravity, the pressure in the cold matter decreases away from the AF. The flow is isentropic, so that the density and the sound velocity decrease together with the pressure. The flow velocity v in this case increases, since the mass flow must be constant and correspondingly $M = v/c$ increases. The appearance of an internal supersonic zone in the stationary flow does not correspond to the essence of the problem of acceleration of the layer by the ablation pressure. For this reason the results of [6, 7] are useful only for $\lambda \ll L_1$. When $\lambda \simeq L_1$ the effect of compressibility of the cold matter becomes significant, but in the formulation of [6, 7] this effect is not taken into account correctly. The question of the compressibility and long-wavelength perturbations is analyzed in detail in this paper.

In addition to the works enumerated above, in which the linear stage is studied, interesting studies [7-9] on calculations of nonlinear two-dimensional flows have also been published; [1, 2, 10-12] concern analytical estimates. In [10, 11] it is proposed that a subsonic AF can be replaced by a jump in the deflagration wave. The work in [12] is based on a study of an unstable zone in which the vectors Δp and $\Delta \rho$ are antiparallel. It is assumed that the growing perturbations are spatially localized in this zone. We note that under the usual [3, 4] conditions ($I \simeq 10^{14}$ W/cm^2, Nd laser, layer thickness $L = 1$-4 μm) the thickness Δ_1 of this zone is small ($\Delta_1 \simeq 0.1$ μm). The growing perturbations can have $\lambda \gg \Delta_1$. In this case, the field of perturbations is spatially localized in a layer of thickness $\simeq\lambda \gg \Delta_1$ adjacent to the AF. In this case, the fine structure and the presence of the unstable zone are of no significance, since for such waves the fine structure is "hidden" within the thickness of the line (associated with the thickness of the "slate pencil") marking the perturbed boundary.

The short-wavelength scale of stabilization $\lambda_a = v_a^2/g \simeq M_a^2 L$, where $M_a = v_a/c_s$, v_a is the velocity of the AF relative to the cold matter, the index a indicates ablation, c_s is the sound velocity in the cold matter near AF, and g is the acceleration of the layer, is estimated in [1].

The effect of compressibility is analyzed in [2]. It is shown that for a typical large ratio of densities on the AF the dispersion curve in the case of isentropic gas coincides with the dispersion curve in the case of an incompressible liquid with an arbitrary ratio of the parameters $v_\lambda^2/c_s^2 \simeq \lambda/L$, where $v_\lambda = \sqrt{|g|\lambda}$.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 110-116, September-October, 1985. Original article submitted June 15, 1984.

 0021-8944/85/2605-0702$09.50

This work extends and provides a more rigorous justification of the results of [1, 2], which are concerned with the effect of ablation and compressibility on the development of the Rayleigh–Taylor instability (RTI); here we determine the parameters determining the nature of the instability (I, L, λ) and the values of the parameters I, L, λ for which the problem can be described by a model of a layer bounded by impermeable, isobaric (free) boundary conditions. In this region the instability of the accelerated AF is the RTI. The only complications appearing here are those associated with taking into account the compressibility and the multilayered structure of the system. The model with isobaric and weakly permeable boundaries has also been used before. For example, it was used in [15] in the problem of instability. The purpose of this work is to determine the limits of applicability of the model.

We shall make a few remarks regarding applications. In applications (see, for example, [3]), shells are studied which are required in order to convert the largest possible relative fraction of the absorbed energy into the kinetic energy of the shell and then, during the soft recuperation, of the kinetic energy stored in the shell into the internal energy of the gas contained inside the shell. It is well known [3] that effective recuperation requires the use of thinner and more massive shells and the acceleration of the shell must be carried out in substantially subsonic states of ablation. The corresponding conditions have the form

$$L \ll R; \tag{1.1}$$

$$\rho_f \ll \rho_s; \tag{1.2}$$

$$v_a \ll c_s, \tag{1.3}$$

where R is the radius of the shell and ρ_s, ρ_f are the density of the cold matter in the vicinity of the AF and of the internal gas.

The acute problem of the hydrodynamic instability arises precisely in connection with the requirements (1.1)-(1.3). Indeed, when (1.3) is retained in thick ($L \lesssim R$) or a weakly degenerate ($\rho_f \lesssim \rho_s$) shell, this question loses its acuteness. The question of the instability also disappears in the case of the supersonic state of propagation of the electronic thermal wave along the shell material, when $v_a \gtrsim c_s$.

The instability is associated either with the AF and develops at the stage when the shell is driven toward the center, or with the internal boundary of the shell and develops at the stage of retardation. As regards the stage of retardation, here the classical Rayleigh–Taylor formulation with an impermeable boundary does not give rise to any objections. We discuss below only the instability at the acceleration stage.

If we restrict our attention to the case $\lambda \ll R$, then the curvature of the shell can be neglected, and the problem of the instability of the shell becomes equivalent to the problem of the instability of a flat layer. Perturbations with $L \ll \lambda \simeq R$ can be effectively studied in the approximation of a liquid film [16].

2. Qualitative Analysis. We shall confine our attention to the case $\lambda > \Delta$. Here and below Δ is the distance from the point at which the value A = 1.1 is reached up to the point at which $A = 10^{\Gamma}$, where Γ is the adiabatic index; the dimensionless variable A, associated with the entropy, is defined below. We note that the value of Δ is equal in order of magnitude to the distance from the maximum of the density up to the point at which the density drops down to $\rho = \rho_s/10$ (see Fig. 1). In addition, Δ is approximately equal to the maximum value of the function $(|\partial \ln \rho(y, t)/\partial y|)^{-1}$.

In studying modes with $\lambda > \Delta$, it is possible to work with the boundary layer $y = \eta(x, t)$ (see Fig. 1).

If the ablation pressure is "switched on" rapidly enough, the initially stationary matter is put into motion by a shock wave with constant intensity. In this case [14, 17, 18], the entropy distribution in the cold matter in the layer is nearly uniform, and it remains uniform also during the acceleration of the layer, since shock waves do not arise in the layer when the ablation pressure p_a is constant, while the electronic thermal conductivity in the cold matter is low and the heat fluxes are correspondingly negligibly small, so that the flow of cold matter may be regarded as adiabatic. The entropy in the liquid part begins to grow when this part intersects the surface of the AF.

To determine the surface of the AF, which we shall call the boundary of the layer, we shall study a system of instantaneous isentropic lines: $S = \text{const} = AS_0$, where the entropy is the quantity $S = p\rho^{-\Gamma}$, A is the dimensionless variable, S_0 is the value of the entropy in

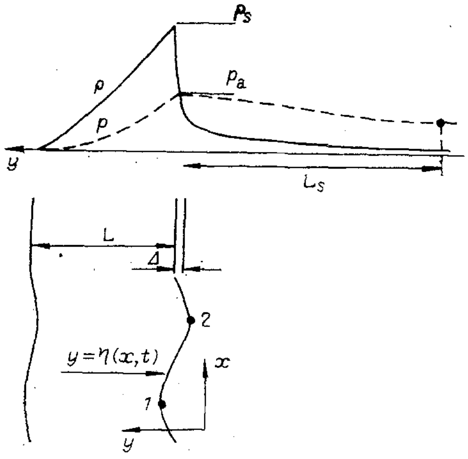

Fig. 1

the cold matter. The isentropic lines corresponding to the values A > 1, drawn with some step $\Delta A > 0$, bunch up in a layer with thickness of the order of Δ. As the surface of the AF we shall choose some isentropic line with $A = A_*(1.1 < A_* < 10^{\Gamma})$, falling into this bunch.

Jumping ahead, we shall present first the final results, obtained with the help of a qualitative analysis. At a fixed radiation frequency the parameters determining the nature of the instability are I, L, and λ. The region of applicability of the model in the space (I, L, λ) has the form

$$I_L(L) \ll I < I_U; \tag{2.1}$$

$$\lambda_{\min} < \lambda \ll L_s(I). \tag{2.2}$$

The Mach number in the corona increases monotonically away from the AF into the corona. We shall denote by L_s the distance from the AF to the acoustic surface on which M = 1 (the index s denotes sound). The quantity L_s depends at a fixed frequency of the radiation solely on the intensity I:$L_s = L_s(I)$. We denote by $I = I_L(z)$ the function inverse to the monotonic function $z = L_s(I)$. The function $I_L(z)$ is a monotonically increasing function of z. It follows from what was said for the functions $I_L(L)$ and $L_s(I)$ that the restrictions $I_L(L) \ll I$ and $L \ll L_s(I)$ are equivalent. Thus the restriction $I_L(L) \ll I$ on the intensity from below (the index L means lower) shows that the model is applicable if the accelerated cold layer is much thinner than the thickness of the subsonic "pillow" L_s. In addition, as will be shown below, the model is applicable if $\lambda \ll L_s(I)$.

The restriction $I < I_U$ on the intensity from above (the index U indicates upper) by some constant I_U independent of L and λ follows from the condition that the motion of the AF along the cold matter must be subsonic. Namely, $v_a(I)$ must be $<c_s(I)$, where $c_s(I) = \sqrt{\Gamma p_a(I)/\rho_s}$ is the sound velocity in the cold matter in the vicinity of AF, and p_a is the ablation pressure. Otherwise, the instability of the AF will be suppressed. The ablation velocity $v_a(I)$ increases with I more rapidly than $c_s(I)$, so that the function $M_a(I) = v_a/c_s$ increases monotonically with I and there exists a value of the intensity I_U at which the value $M_a(I_U) = 1$ is attained.

The lower limit $\lambda_{\min}$ is less than or approximately equal to the largest of the quantities Δ and λ_a, where $\Delta = \Delta(I)$, $\lambda_a = \lambda_a(I, L) = v_a^2/|g| = (\Gamma - 1)\ M_a^2 L$. The fact that the function λ_a(I, L), determining the boundary of the region of applicability of the model with respect to small-scale modes, is equal to $v_a^2/|g|$ follows from the hypothesis 2 [formula (2.4)] and the condition (2.7), under which the boundary of the layer may be regarded as impermeable.

We note that if the values of I and L belong to the projection of the region of applicability of the model onto the (I, L) plane, then it follows from the conditions (2.1) and (2.2) that the mode $\lambda \cong L$ belongs to the region of applicability of the model. Indeed, let I and L belong to the projection. This means that the two conditions $L \ll L_s$ and $v_a \ll c_s$ hold, whence it follows that, first of all, $\lambda \ll L_s$ because $\lambda \simeq L \ll L_s$ and, second, $\lambda > \lambda_{\min}$. The latter condition $\lambda > \lambda_{\min}$ is obtained as follows. We have $L > \lambda_{\min}$. Indeed, $L > \Delta$ and $L > \lambda_a = (\Gamma - 1)M_a^2 L$ when $M_a < 1$ (usually $M_a \ll 1$), and this means that $\lambda \simeq L > \lambda_{\min}$ also.

In connection with the applications in [3], intensities of $I \simeq 10^{14}$ W/cm^2 (Nd laser) are studied. In this case [3, 14], $v_a = (0.1\text{-}0.3)c_s$, L_s = 20-30 µm, $\Delta \simeq 0.1$ µm, L = 1-4 µm.

As we can see, for these values of I and L the conditions (2.1) hold.

A qualitative analysis leading to (2.1) and (2.2) rests on two assertions.

Assertion 1 (regarding "suddenness" of the response). In the system of coordinates tied to the unperturbed motion of the layer (the unperturbed cold matter moves as a whole [14, 17, 18]), the boundary moves with a velocity $\simeq v_a$ or in the region of applicability of the model with a velocity $\simeq v_\lambda = \sqrt{|g|\lambda}$. Both these velocities are much lower than the characteristic velocity scale in the corona. Based on this, it may be asserted that the response of the pressure p_a and velocity v_a in the corona to the instantaneous form of the boundary is "instantaneous." The speed of the response is associated with the higher mechanical and thermal inertia of the layer than of the corona, owing to the high density of the cold matter, its low thermal conductivity, and its high heat capacity per unit volume.

In accordance with this assertion p_a and v_a are some functionals of η (but not of $\partial\eta/\partial t$). We shall represent η in the harmonic form $\eta = \eta_0 + \delta\eta \sin kx$. In what follows we shall work with the small amplitude $\delta\eta$. In this case, we have $p_a = p_{a0} + \delta p_a \sin kx$, $v_a = v_{a0} + \delta v_a \sin kx$ (the phases coincide, since there are no oscillatory effects), $\delta p_a = G_d\delta\eta$, $\delta v_a = G_k\delta\eta$. Here $\delta\eta = (\eta_1 - \eta_2)/2$; $\delta p_a = (p_{a1} - p_{a2})/2$; $\delta v_a = (v_{a1} - v_{a2})/2$; the indices 0, 1, 2 in front of η, p_a, v_a refer, respectively, to the unperturbed state, the trough, and the bulge in the boundary relative to the layer (see Fig. 1). The responses G_k and G_d (we call them the "kinematic" and "dynamic" responses) are required in order to estimate the breakdown in the degree of impenetrability (see the hypothesis 2) and the degree of isobaricity (see hypothesis 1) at the boundary of the cold matter.

Assertion 2. The structure of the corona in the case of a not very thin layer with $L > L_{min} \simeq 0.3$ μm does not depend on L. Correspondingly, the dependence on L drops out of the functions $p_a(I)$, $v_a(I)$, $L_s(I)$, $c_s(I)$, $G_d(\lambda, I)$, $G_k(\lambda, I)$.

This means that G_d and G_k can be evaluated for $L = \infty$.

The results (2.1) and (2.2) follow from two hypotheses.

Hypothesis 1. The modes of interest to us are localized near the AF in a layer whose thickness is equal in order of magnitude to λ. For $\lambda \ll L_s$, when the localization of the perturbations deep in the corona is limited by the substantially subsonic section of the corona, a rough (apparently with some margin) estimate of the upper limit on the amplitude of the pressure response has the form

$$|\delta p_a|/\delta\eta = |G_d| < p_a/L_s. \tag{2.3}$$

Hypothesis 2. It is easy to see that the "kinematic" response $G_k < 0$, since more cold matter must flow per unit time through a unit surface area of the AF from the bulge in the AF than from the trough in the AF (see Fig. 1, the troughs and bulges relative to the cold matter are denoted by the numbers 1 and 2, respectively), and can be estimated as follows:

$$(-G_k) < v_a/\lambda \quad \text{or} \quad (-G_k) \simeq v_a/\lambda, \tag{2.4}$$

since the opposite inequality $(-G_k) \gg v_a/\lambda$ is impossible for $\lambda > \Delta$. To clarify the essence of Hypothesis 2, we shall examine the perturbations of the AF with a large amplitude $2\delta\eta = \eta_1 - \eta_2 \simeq \lambda$. An order of magnitude estimate of G_k is suitable in this case, of course, for $\delta\eta \ll \lambda$ also. Hypothesis 2 essentially follows from the assertion that the velocity v_a, equal to the normal (relative to the surface of the AF) component of the velocity of the cold matter with which the liquid particle of cold matter approaches the AF, is determined primarily by the intensity I. The latter fact indicates that in the case of the perturbed ("undulating") surface of the AF the velocity v_a, though it is different at different points on the surface of the AF, is still determined in order of magnitude as before, and as in the unperturbed case, by the value of I, i.e., the estimate $v_{a1} \simeq v_{a2} \simeq v_{a0}$ will also hold when $\delta\eta \simeq \lambda$. This is the basis of Hypothesis 2.

For what follows we shall require a formula relating g and L. From the equation of hydrostatics and the condition of continuity of the pressure on the AF $p_s = p_a$ we have

$$|g| = \frac{\Gamma p_a}{(\Gamma - 1)\rho_s L} = \frac{c_s^2}{(\Gamma - 1)L}. \tag{2.5}$$

The numerical coefficient in (2.5) was obtained for an isentropic layer and with the condition p = 0 on the back side.

We shall assume that the pressure distribution is isobaric, if

$$|G_d| \ll \rho_s |g| = \Gamma p_a / [(\Gamma - 1)L]. \tag{2.6}$$

The conditions (2.3) and (2.6) can be put into the form $L \ll L_S$. From here follows the left side of (2.1) and the right side of (2.2).

The boundary is assumed to be impenetrable, if

$$-G_k \ll v_\lambda/\lambda,\ v_\lambda = \sqrt{|g|\lambda}. \tag{2.7}$$

Combining (2.4) and (2.7), we find that for $\lambda > \lambda_a$ $v_\lambda > v_a >$ or $\simeq G_k|\lambda$, where λ_a is given by the formula $\lambda_a = v_a^2/|g|$. The condition $\lambda > \lambda_a$ with $\lambda_a > \Delta$ gives the left side of (2.2).

It remains to study the case when $\lambda_a \ll \Delta$. In this case, $v_a \ll \sqrt{\Delta|g|}$, $\gamma_*^{-1} \ll \Delta/v_a$, and the ablation pressure can be neglected. We thus arrive at the problem of the RTI in a static layer with a finite density gradient studied previously (see, for example, [19]). In this case the increments of all possible RT modes are limited from above by the value $\simeq\gamma_*$, where $\gamma_* = \sqrt{|g|/\Delta}$ is the Brunt–Väisälä "increment."

At the nonlinear stage of the development of the instability, in the region of applicability of the model, the rate of loss of mass by the layer $\dot{\sigma}_\lambda = \rho_S v_\lambda'$ due to the RTI must be compared with the rate of ablation losses $\dot{\sigma}_a = \rho_S v_a$. It is well known [20] that $v_\lambda' = Fr\sqrt{|g|\lambda}$, where Froude's number Fr = 0.23-0.40. The value 0.23 is obtained for a two-dimensional flow in the form of levees, and the value 0.40 corresponds to a square lattice. As a result we arrive at a scale $\lambda_a' = (Fr)^{-2}\lambda_a$ larger than λ_a. A nonlinear quasistationary flow can apparently be established in the region $\lambda_a < \lambda < \lambda_a'$. Such a flow was observed in [7].

The study of the pressure response p_a leads to the conclusion that the model is inapplicable when $L \simeq L_S$. The conclusion that in the case of a thick subsonic "pillow" $L \ll L_S$ isobaric conditions will exist for perturbations with $\lambda <$ or $\simeq L$ appears to be quite convincing. This is also supported by the two-dimensional calculation in [7], in which for values satisfying the conditions $L \ll L_S$, $v_a \ll c_S$ a flow pattern which is typical for RTI was obtained.

3. Isobaric RT Mode. Let us examine the case of the most dangerous perturbations with $\lambda \simeq L$. If the values of I and L belong to the projection of the region, then the modes $\lambda \simeq L$, as shown in Sec. 2, fall into the region of applicability of the model. From (2.5) we obtain $v_\lambda^2/c_S^2 = (\lambda/L)(\Gamma - 1)^{-1}$. This means that for $\lambda \simeq L$ the compressibility of the cold matter must be taken into account. In addition, the multilayer case is of interest in applications.

It can be shown that the motion

$$z = \zeta + A \exp(ik\zeta^* + \gamma t),\ \gamma^2 = -k|g|, \qquad p = -|g| \int^{b=b(x,y,t)} \rho(y_1)\, dy_1 \tag{3.1}$$

is the exact solution of the linearized equations of gasdynamics in an arbitrarily layered compressible liquid with an arbitrary value of the parameter λ/L. The corresponding motion is the motion with "frozen in" isobars, in which the values of the pressure are preserved in the liquid particles. It follows from this property that (3.1) satisfies the free boundary conditions. The following notation is used in the formulas (3.1):

$$z = x + iy,\ \zeta = a + ib,\ \zeta^* = a - ib, \qquad b = y - |A| \exp(ky + \gamma t) \sin(kx + \delta),\ A = |A| e^{i\delta} \tag{3.2}$$

(x, y are Eulerian and a, b are Lagrangian coordinates), where $\rho(y)$ is the variation of the density in the unperturbed liquid, and the y axis is oriented opposite to the vector **g**. We obtain isobaric damped and growing Rayleigh–Taylor modes for $k < 0$ and two isobaric gravity waves for $k > 0$. The expression for $b = b(x, y, t)$ in (3.2) is presented for $k < 0$, when the values of γ are real.

In the particular case of a homogeneous incompressible liquid the solution with $\gamma^2 = -k|g|$ was found in [21]. It is shown in [2, 22] that when $\gamma^2 = -k|g|$ the modes exist in a flat isentropic layer. It is shown in [22] that the mode with the increment $\gamma = \sqrt{-k|g|}$ ($k < 0$) is the only growing mode in the isentropic layer. In [23] the result of [21] is extended to a layered incompressible liquid. In this work it is emphasized that the solution is isobaric and is applicable to an arbitrarily layered compressible liquid.

The isobaric mode solves the problem of multilayeredness and compressibility in the problem of rupture of the layer. The point is that this mode has the maximum increment amongst all possible RT modes. Therefore, rupture occurs precisely because of the growth of the mode with $\gamma = \sqrt{-k|g|}$ (k < 0). The lifetime of the layer is determined by the spectrum of the initial perturbations [24] and the thickness of the layer. A pronounced layeredness decreases the lifetime, since for a fixed total thickness the effective thickness, equal to the thickness of the interlayer with the highest density containing a relative fraction of ≅1 of the total mass of the layer, decreases.

In the case of two stationary gases with the same adiabatic indices, it is possible to calculate (the calculations are cumbersome and are not presented here), using the methods described in [2], the correction to the isobaric mode, associated with the finite magnitude of the ratio of the gas densities on the contact boundary. The dispersion curve has the form

$$\gamma^2 = k|g| - \frac{\mu|g|}{L}\frac{(\beta+1)e^{2kL}}{F(\beta+1, \beta+2, 2kL)} + O(\mu^2),$$

where L is the thickness of the top layer; $\mu = \rho_H/\rho_B < 1$; ρ_H, ρ_B are the density of the bottom and top gases, respectively, on the contact boundary; $\beta = (\Gamma - 1)^{-1}$; and $F(\alpha, \beta, z)$ is the confluent hypergeometric function.

In conclusion, the author thanks S. I. Anisimov for his constant interest in this work.

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ANALYSIS OF THE EFFECT OF THERMOELASTIC STRESSES OF THE CRYSTALLIZATION OF A SPHERE UNDER WEIGHTLESS CONDITIONS

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UDC 539.319

In the crystallization of a sphere initially completely molten and then cooled slowly over its entire surface, thermoelastic stresses are created in the solid phase. If the intensity of the shear streasses reaches the critical value – the yield point – a region of plastic deformation appears. A description was given in [1] of experiments involving crystallization of copper and silver specimens in the form of spheres (the amount of impurity was 0.001% in the copper specimen and 0.004% in the silver specimen) under weightless conditions. It was noted that the structure of the specimens obtained indicates a nearly complete lack of convective motion in the melt. It is interesting to study the effect of thermoelastic stresses on the crystallization of specimens under weightless conditions and on the structure of the crystals obtained.

The study [2] indicated that it might be possible to form a shrinkage cavity during the crystallization of a sphere if the solid phase is denser than the liquid phase. The occurrence of thermoelastic stresses is one possible cause of shrinkage cavity formation. In the model in [2], the cavity begins to form at the very beginning of the crystallization process. Thus the stresses in the solid phase are due only to incompatible thermal strains, not to shrinkage of the material, and it can be anticipated that the resulting stresses will not have an appreciable effect on subsequent crystallization.

Here we study the process of crystallization without the formation of a shrinkage cavity.

1. The material is assumed to be incompressible in the liquid state and it is assumed that the crystallization process occurs in the absence of external effects (under weightless conditions and in vacuum).

The crystallization process was studied numerically for metals (copper, aluminum, silver) in [3] and for semiconducting materials (germanium, silicon) in [4]. The problem was formulated in an isotropic approximation for all of the materials.

We introduce a spherical coordinate system r, φ, θ with its origin at the center of the sphere. We have the following relations for the liquid phase:

$$\frac{\partial T_2}{\partial t} = \frac{\lambda_2}{\rho_2 c_2}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T_2}{\partial r}\right); \tag{1.1}$$

$$P = -p(t)I, \tag{1.2}$$

where T_2, ρ_2, c_2, and λ_2 are the temperature, density, specific heat, and thermal conductivity of the liquid phase; P is the stress tensor; I is a unit tensor.

The behavior of the material in the solid phase is described by a system of thermoelastoplasticity equations. Due to spherical symmetry, only the normal components of the stress tensor σ_r, σ_φ, σ_θ, are nontrivial, while $\sigma_\varphi = \sigma_\theta$. The following equilibrium equation holds throughout the region of the solid state

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 117-122, September-October, 1985. Original article submitted September 18, 1984.

 0021-8944/85/2605-0708$09.50